

Emergence of Fluctuations from a Tachyonic Big Bang

Robert H. Brandenberger,^{*} Andrew R. Frey,[†] and Sugumi Kanno[‡]
Department of Physics, McGill University, Montréal, QC, H3A 2T8, Canada
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It has recently been speculated that the end state of a collapsing universe is a *tachyonic big crunch*. The time reversal of this process would be the emergence of an expanding universe from a tachyonic big bang. In this framework, we study the emergence of cosmological fluctuations. In particular, we compare the amplitude of the perturbations at the end of the tachyon phase with what would be obtained assuming the usual vacuum initial conditions. We find that cosmological fluctuations emerge in a thermal state. We comment on the relation to the *trans-Planckian problem* of inflationary cosmology.

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I. INTRODUCTION

There have recently been several proposals to study the resolution of cosmological singularities in string theory. Postulating a tachyon condensate phase in the very early universe [1] is one of these new avenues. In this paper, we make a first attempt to connect such an early stringy phase of cosmology with cosmological observations. Specifically, we study the origin of cosmological fluctuations in this framework.

If the initial tachyon condensate phase of string cosmology is followed by a period of cosmological inflation, then the spectrum of fluctuations produced in the initial phase will be observable via the exponential redshift of the wavelength which the perturbation modes undergo. Thus, the initial string phase of cosmology will set the initial conditions for the fluctuation modes in the ultraviolet regime at the beginning of the period of inflation.

In this way, our study connects to one of the conceptual problems [2] of inflationary cosmology, namely the *trans-Planckian problem*. The issue is the following: provided that the period of inflation lasted more than 70 e-foldings, then the physical wavelengths of all fluctuations which are currently observed have a physical wavelength at the beginning of the period of inflation which is smaller than the Planck scale. Hence, as first studied explicitly in [3], the predictions of the theory for observations are sensitive to the unknown ultraviolet physics which determines the evolution of fluctuations at early times in the ultraviolet (UV) region. In the context of scalar field-driven inflation, there have been several approaches to deal with this problem - making use of dispersion relations which are modified in the UV region [3, 4], incorporating the effects of space-space [5] or space-time [6] non-commutativity, appealing to a minimal length [7], assuming a “new physics hypersurface” [8], or making use of effective field theory techniques [9] (see e.g. [10] for a

review and for more references). [29]

The key point is that, since the cosmological fluctuations simply red-shift in an expanding universe, the imprints of trans-Planckian physics are not diluted but simply red-shift. Thus, imprints of trans-Planckian physics are, in principle, measurable in current observations. The *trans-Planckian problem* for inflationary cosmology has become the *trans-Planckian window of opportunity* to probe Planck-scale physics today. To realize this window of opportunity, however, it is crucial to correctly couple the cosmological fluctuations to the fundamental theory which describes UV physics.

In this paper, we will address this question in the context of a recent perturbative approach to singularity resolution in string theory via the condensation of closed string tachyons [1]. To be specific, we will consider the setup of [11] in which the condensation of closed string winding modes is considered in a contracting homogeneous universe. Once the radius of space decreases below a critical value, the winding modes become tachyonic, and a tachyon condensate forms. As the tachyon condensate increases in magnitude, all string fluctuations, including those which correspond to cosmological perturbations, are frozen out. One interpretation of this phenomenon is that the dimensions of space in which the tachyon condenses disappear, and that thus a topology change occurs (see e.g. [12] for further references on this interpretation). Here, we will consider the time reverse of this dynamics: our universe is emerging from a tachyonic big bang. As the tachyon condensate decreases, fluctuation modes gradually emerge. We will track these fluctuations from the time when they emerge until the time t_0 when the tachyon condensate has disappeared, and we will compare the result with what would be obtained from naive vacuum initial conditions of the low energy field theory fluctuations at the time t_0 .

We find that in our setup, cosmological fluctuations emerge in a thermal state. If the early tachyonic phase of cosmology connects to the late time universe via a phase of inflation, and the period of inflation is comparable to the minimal period required for inflation to solve the cosmological puzzles of standard big bang cosmology, then we obtain a thermal rather than a scale-invariant spec-

^{*}email: rhb@hep.physics.mcgill.ca

[†]email: frey@hep.physics.mcgill.ca

[‡]email: sugumi@hep.physics.mcgill.ca

trum of fluctuations. Only if inflation lasts much longer than the minimal number of e-foldings, then scales being probed in observations today would correspond to the far Wien tail of the thermal spectrum, and thus the predictions for observations would be similar to what would be obtained assuming the usual vacuum state for cosmological perturbations. If the initial tachyon phase connects to our present universe via the recently suggested string gas structure formation scenario, the tachyonic phase studied in this paper could provide the required thermal initial conditions.

The outline of this paper is as follows: in the next section we will derive the equations of motion for cosmological perturbations which hold given a period of tachyon condensation. In Section 3, we will solve these equations and compute, making use of the Bogoliubov mode mixing technique, the spectrum of perturbations at the end of the tachyon phase given an initial ground state. We then discuss the implications of our results for cosmology and conclude with a summary section.

II. EQUATIONS OF MOTION FOR THE FLUCTUATIONS

In this section we will derive the equation of motion for cosmological fluctuations which hold in the weak tachyon period of tachyon condensation (see figure 1). Our approach will be to consider two different string backgrounds as toy model cosmologies and find that the equations of motion for fluctuations share the same features. We will work in the string frame and in tree-level (flat worldsheet) string perturbation theory throughout.

A. Tachyon Condensation and Dimension Change

Even though we imagine a cosmology such as that of [11] in which the closed string tachyon is localized to a region of spacetime with a small circle, we will find it simpler to consider models in which the tachyon is present throughout the entire spacetime, as in [12]. In this case, the spacetime metric is flat at all times, but the dimensionality of spacetime is reduced in regions of large tachyon. Therefore, some (or all) of the spacetime is in a noncritical dimension, so there must be a dilaton gradient. We will choose the dilaton gradient to be timelike in the region of smaller tachyon (and larger dimensionality), and we will proceed to ignore the dilaton gradient because it will modify the equations of motion only by a “Hubble-damping/forcing” type term, and we prefer to focus on other aspects of the physics.

From the point of view of the 2D worldsheet theory, the dimensions of spacetime are just quantum fields, and the tachyon condensate induces a (spacetime-dependent) mass for some of these fields. Therefore, the worldsheet field theory flows to a target spacetime of lower dimension as the massive fields are integrated out; most work has

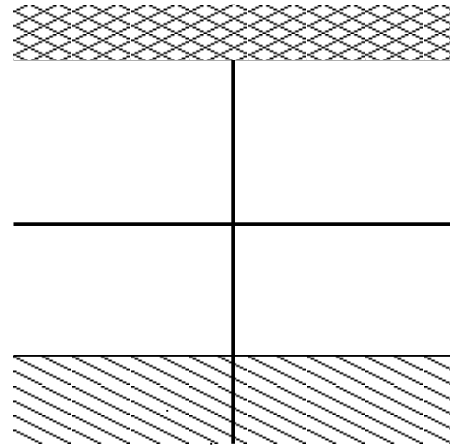


FIG. 1: Space-time sketch of our tachyon condensate background. The vertical axis denotes time, the horizontal axis is space. The bottom dashed region indicates the region of strong tachyon condensation when it does not make sense to talk about space. The upper cross-hatched region is the region of strong coupling due to the linear dilaton, which we assume is stabilized to yield a standard cosmology. Our analysis studies the emergence of fluctuations in the intermediate un-hatched weak tachyon phase.

been concerned with arguing for that RG flow [11, 12]. (Note that this argument requires that the tachyon itself depend on the spatial dimensions which it eliminates, or else those dimensions would remain massless as worldsheet fields.) On the other hand, we are interested in the spacetime emerging from the tachyon condensate, so we will study the conformal interacting worldsheet theory relevant for regions of spacetime with a small tachyon. To keep matters simple, we will work with the bosonic string (or equivalently the bosonic half of a nonsupersymmetric heterotic string). Additional details and further results on this theory from the perspective of the small tachyon region will be presented in [13].

B. Backgrounds and Gauges

As we mentioned above, the spacetime in the string backgrounds we consider are flat (in the string frame), and we imagine the tachyon to have a spacetime profile of the form

$$T(X^\mu) \propto e^{-2\beta X^0} \vec{X}^2 \quad (1)$$

or

$$T(X^\mu) \propto e^{-2\beta X^+} \vec{X}^2. \quad (2)$$

Here, \vec{X} runs over some of the spatial dimensions (which we take for simplicity to be all the transverse spatial dimensions), which are the spatial dimensions destroyed in the region of large tachyon condensate, and β controls the gradient (and is generally related to the dilaton gradient). These profiles are appropriate for the

bosonic string theory (up to the addition of an additional space-independent term which is unimportant for our purposes); to get the same worldsheet action for the heterotic string, we take $T \rightarrow T^2$ in the profiles (1,2). Specifically, in either case, the worldsheet action for the string (ignoring the dilaton) will be

$$S = -\frac{1}{4\pi\alpha'} \int_0^\ell d\tau d\sigma \left[\partial_a X^\mu \partial^a X_\mu + \frac{\mu^2}{2} e^{-2\beta t} \vec{X}^2 \right], \quad (3)$$

where $t = X^0$ or $t = X^+$ respectively and ℓ is the coordinate length of σ on the worldsheet (with periodic boundary conditions). Note that the tachyon background does introduce other terms in the action (depending only on t in the bosonic string and involving fermions in the heterotic string), but these will not affect our analysis of the bosonic modes.

From action (3), it seems that the obvious gauge choice would be to take $t = \alpha' \hat{p} \tau$, relating spacetime and worldsheet times (where $\hat{p} = p^{0,+}$ for shorthand). To get to this gauge, we first take worldsheet coordinates such that the worldsheet metric is conformally flat. In these coordinates, t satisfies a wave equation on the worldsheet, so we can write $t = f(\bar{\tau} + \bar{\sigma}) + g(\bar{\tau} - \bar{\sigma})$. Then we can take $\tau = t$ and $\sigma = f(\bar{\tau} + \bar{\sigma}) - g(\bar{\tau} - \bar{\sigma})$, which leaves the metric conformally flat. Because the theory is conformally invariant, we can just use the flat worldsheet metric. As a final note, we can set the worldsheet spatial coordinate periodicity to $\ell = 2\pi$ by concurrently rescaling the tachyon amplitude μ (or equivalently shifting $X^{0,+}$).

There are, however, caveats in the two cases. First, for the timelike tachyon, this gauge is only sensible if we treat the tachyon contribution to the action as a perturbation and set the zeroth order part of t proportional to τ . This follows because higher order parts of X^0 are not harmonic on the worldsheet according to their equations of motion. Also, in order to avoid α' corrections to the tree-level quantization, we need small β and therefore a large positive dilaton gradient. This means that string theory becomes strongly coupled in the future. We will deal with this objection by supposing the strong coupling region to be in the far future, after the tachyon generates fluctuations. In a more realistic model, we would also expect the dilaton to stabilize at some finite value.

For the null tachyon gradient, we no longer have to worry about α' corrections, as discussed in [12]. However, while we have chosen a lightcone gauge, the dilaton gradient is still timelike in a supercritical number of dimensions. To circumvent this difficulty, we choose to have critical dimension in the weak tachyon region. In this case, the tachyon profile (2) solves the equations of motion for a lightlike dilaton $\Phi \propto X^-$, which does not interfere with the lightcone quantization. Note that there is again a strongly-coupled region of spacetime (both due to the lightlike dilaton in the weak tachyon region and a spacelike dilaton gradient in the strong tachyon region). However, again, we can push the strong-coupling region arbitrarily far away, so we expect that it does not affect our results.

C. Worldsheet Hamiltonian

To quantize the string, we first need to work out the mode expansion of the transverse coordinates \vec{X} . To first order in the tachyon background, we find

$$\begin{aligned} \vec{X} = & \vec{x} \left(1 - \frac{1}{4} \left(\frac{\mu}{\alpha' \beta \hat{p}} \right)^2 e^{-2\alpha' \beta \hat{p} \tau} \right) \\ & + \alpha' \vec{p} \tau \left(1 - \frac{1}{2} \left(\frac{\mu}{\alpha' \beta \hat{p}} \right)^2 (1 + \beta t) e^{-2\alpha' \beta \hat{p} \tau} \right) \\ & + i \sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \left[\vec{\alpha}_n e^{-in(\tau+\sigma)} \left(1 - \frac{1}{4(1+\nu)} \left(\frac{\mu}{\alpha' \beta \hat{p}} \right)^2 e^{-2\alpha' \beta \hat{p} \tau} \right) + \vec{\alpha}_n e^{-in(\tau-\sigma)} \right. \\ & \left. \times \left(1 - \frac{1}{4(1-\nu)} \left(\frac{\mu}{\alpha' \beta \hat{p}} \right)^2 e^{-2\alpha' \beta \hat{p} \tau} \right) \right] \end{aligned} \quad (4)$$

where $\nu = in/\alpha' \beta \hat{p}$. This is the usual mode expansion with an exponentially dying correction for each mode (including the center of mass coordinate). To avoid working with worldsheet fermions, we specialize to the bosonic string.

There are two ways to derive (4). One is to note that the equation of motion

$$(-\partial_\tau^2 + \partial_\sigma^2) \vec{X} = \mu^2 e^{-2\alpha' \beta \hat{p} \tau} \vec{X} \quad (5)$$

has for each spatial mode $\exp(in\sigma)$ the solution

$$\vec{X}_n = \vec{a}_n J_\nu \left(\frac{\mu}{\alpha' \beta \hat{p}} e^{-\alpha' \beta \hat{p} \tau} \right) + \vec{b}_n Y_\nu \left(\frac{\mu}{\alpha' \beta \hat{p}} e^{-\alpha' \beta \hat{p} \tau} \right). \quad (6)$$

Then we can simply expand around late times and match the leading terms to the usual mode expansion. Alternatively, we can split the equation of motion (5) and oscillator \vec{X} into zeroth and first order parts, which gives exactly the correction terms in (4) as the first order part of (6).

From the mode expansion (4), we can write out the worldsheet Hamiltonian in our two gauges as

$$\begin{aligned} H = & \frac{1}{2} \alpha' \vec{p}^2 \left(1 + \left(\frac{\mu}{\alpha' \beta \hat{p}} \right)^2 (1 + \beta t)^2 e^{-2\beta t} \right) \\ & + \sum_{n=1}^{\infty} \left[(\vec{\alpha}_{-n} \cdot \vec{\alpha}_n + \vec{\alpha}_{-n} \cdot \vec{\alpha}_n) \left(1 + \frac{\mu^2}{4n^2} e^{-2\beta t} \right) \right] - 2 + \left\{ \frac{\alpha' \hat{p}^2}{2} \right\} + \dots, \end{aligned} \quad (7)$$

in which the term in curly braces only appears for the static gauge case. Also, in the static gauge case, we need to add the usual momentum term without corrections for the longitudinal direction (usually X^1), but not the oscillators (because they are not independent degrees of

freedom). We have denoted by ellipses terms involving the center of mass position and terms which change the state of the string. For our first analysis, we will ignore these effects. Finally, there are terms correcting the worldsheet zero-point energy. Considering the exact results of [12] in the lightlike tachyon case, we know that these corrections to the zero-point energy must vanish once the linear dilaton and all quantum effects have been taken into account.

D. Model Equation of Motion

To relate the worldsheet Hamiltonian to the physics of fluctuations in spacetime, consider the worldsheet Schrödinger equation

$$H = i\partial_\tau = i\alpha' \hat{p} \partial_t. \quad (8)$$

This last derivative is just the momentum conjugate to t , either ip_0 or ip_+ in the timelike and lightlike case respectively. Then we have

$$H = \alpha' \times \begin{cases} (p^0)^2 \\ \text{or} \\ p^+ p^- \end{cases}. \quad (9)$$

Comparing (7,9), we can write the Schrödinger equation as a mass-shell relation,

$$0 = -(p^0)^2 + \vec{p}^2 \left(1 + \left(\frac{\mu}{\alpha' \beta p^0} \right)^2 (1 + \beta t)^2 e^{-2\beta t} \right) + \frac{\mu^2}{2\alpha'} e^{-2\beta t} \quad (10)$$

or

$$0 = -2p^+ p^- + \vec{p}^2 \left(1 + \left(\frac{\mu}{\alpha' \beta p^+} \right)^2 (1 + \beta t)^2 e^{-2\beta t} \right) + \frac{\mu^2}{2\alpha'} e^{-2\beta t}. \quad (11)$$

We have specialized here to the case of massless string modes (such as the dilaton and graviton), which have $\langle \vec{\alpha}_{-1} \cdot \vec{\alpha}_1 \rangle = \langle \vec{\tilde{\alpha}}_{-1} \cdot \vec{\tilde{\alpha}}_1 \rangle = 1$ and all other oscillators unexcited.

It is worth commenting on the corrections to the Minkowski mass-shell relation $p^2 = 0$. The simplest term is the common last term of (10,11), which is just a time-dependent mass term for the string mode. This term arises because the \vec{X} oscillators have a time-dependent excitation energy on the worldsheet. From a spacetime point of view, this mass term must arise from a graviton-tachyon coupling. The other correction visible in (10,11) is a modified dispersion relation, which essentially corresponds to a time-varying speed of light. Some points need to be made about this term. First, the time-dependence is no longer a pure exponential. However, the correction

is subleading, so we will ignore it for simplicity. Second, the coefficient of the correction contains either $1/(p^0)^2$ or $1/(p^+)^2$ in the two cases. This term appears problematic because the momenta should be represented as derivatives in the equation of motion. However, we will treat them (in this correction term only) as constants, for the following reason. In the null tachyon case, p^+ commutes with t and is in fact a constant of motion. Similarly, p^0 is a constant of motion for the timelike tachyon background at zeroth order in the tachyon. Nonetheless, this is a curious dependence, and we will return to this issue in [13].

We close this section with a few comments on the terms we have neglected. First, there are explicitly position-dependent terms in the mass-shell relations. This means that momentum eigenmodes are not actually eigenmodes of the full system; we expect this result because the tachyon varies through space. Next, there are also terms mixing different oscillation states of the string, so the normal Fock space of string excitations is not an eigenbasis. Essentially, the time-varying tachyon allows the string to grow or shrink in time.

III. SOLUTIONS OF THE EQUATIONS

A. Normalized Form of the Equation of Motion

In this section we will discuss approximate solutions of the equation derived in the previous section, focusing on the time-like case due to its relevance to cosmology (the light-like tachyon gives us confidence in our results, however). The starting point is the mass-shell condition (10), which is also the equation of motion for the linearized cosmological perturbation mode ψ in the presence of a time-like tachyon condensate. The Fourier mode of ψ with comoving wave number k satisfies the equation

$$\left[\frac{\partial^2}{\partial t^2} + \left(c_2 \frac{\mu^2}{\alpha'} + c_1 k^2 \right) e^{-2\beta t} + k^2 \right] \psi(t) = 0. \quad (12)$$

Here, c_1 and c_2 are coefficients determining the strength of the corrections to the mass and dispersion relation.

This equation describes the evolution of a harmonic oscillator with a mass term which depends on the tachyon condensate and thus depends exponentially on time. At very early times $t \rightarrow -\infty$, the frequency of oscillation grows exponentially, whereas in the late time limit $t \rightarrow +\infty$, ψ is oscillating with its bare mass. As a consequence of the time dependence of the mass we expect particle production. A mode which is pure positive frequency in the past will be composed of a mixture of positive and negative frequency modes in the future. The mode mixing is given by the Bogoliubov mode mixing matrix.

If we define the following variables

$$-2\beta t \equiv X, \quad \frac{1}{4\beta^2} \left(c_2 \frac{\mu^2}{\alpha'} + c_1 k^2 \right) \equiv \lambda, \quad \frac{k^2}{4\beta^2} \equiv \omega^2, \quad (13)$$

then Eq. (12) can be rewritten in the normalized form

$$\left[\frac{\partial^2}{\partial X^2} + \lambda e^X + \omega^2 \right] \psi(X) = 0. \quad (14)$$

This is the same equation which was studied by Strominger [14] and collaborators [15] in their analysis of open string production by decaying S-branes. To make the correspondence with the equation of motion studied in [15], we must make the substitution

$$\omega^2 = N - 1 + k^2. \quad (15)$$

In contrast to the analysis in [15], in our case we have k dependence in λ . Thus, we can expect that the Bogoliubov coefficients which describe string production in [15] and the production of cosmological perturbations in our case will have a different k dependence.

B. Bogoliubov Coefficients

Let us now calculate the Bogoliubov coefficients in this setup. To describe the initial vacuum state, we must consider the mode which is correctly normalized and pure positive frequency in the past and then expand this mode in the asymptotic future in terms of correctly normalized positive frequency and negative frequency states. The expansion coefficients are the Bogoliubov coefficients we desire. Since the Bogoliubov mode mixing matrix is unitary, we can equivalently take the vacuum state at future infinity and expand it in terms of the correctly normalized positive and negative frequency modes in the far past. This is what will be done below.

Since Eq. (14) can be rewritten by using the variable $y = 2\sqrt{\lambda}e^{X/2}$:

$$\left[\frac{\partial^2}{\partial y^2} + \frac{1}{y} \frac{\partial}{\partial y} + 1 + \frac{4\omega^2}{y^2} \right] \psi(y) = 0, \quad (16)$$

we see that the solutions are Bessel functions: $\psi \propto J_{\pm 2i\omega}(y)$, more precisely,

$$\psi_k^{\text{out}} = \frac{\lambda^{i\omega}}{\sqrt{2\omega}} \Gamma(1 - 2i\omega) e^{i\vec{k} \cdot \vec{x}} J_{-2i\omega}(2\sqrt{\lambda}e^{\frac{X}{2}}), \quad (17)$$

where we chose the above particular solution to be the canonically normalized positive frequency mode in the far future $X \rightarrow -\infty$ (which describes the vacuum state in the future)

$$\begin{aligned} & \frac{\lambda^{i\omega}}{\sqrt{2\omega}} \Gamma(1 - 2i\omega) e^{i\vec{k} \cdot \vec{x}} J_{-2i\omega}(2\sqrt{\lambda}e^{\frac{X}{2}}) \\ & \cong \frac{1}{\sqrt{2\omega}} e^{-i\omega X + i\vec{k} \cdot \vec{x}}, \end{aligned} \quad (18)$$

where we have used the well-known asymptotic form

$$J_\nu(z) \cong \left(\frac{z}{2}\right)^\nu \frac{1}{\Gamma(\nu+1)} \quad \text{for } z \rightarrow 0 \quad (19)$$

of the Bessel functions.

On the other hand, in the far past $X \rightarrow \infty$, the solution (17) becomes

$$\begin{aligned} \psi_k^{\text{out}} & \rightarrow \frac{\lambda^{i\omega-1/4}}{\sqrt{8\pi\omega}} \Gamma(1 - 2i\omega) e^{-\frac{X}{4} + i\vec{k} \cdot \vec{x}} \\ & \left[e^{\pi\omega - 2i\sqrt{\lambda}e^{X/2} + \frac{\pi}{4}i} + e^{-\pi\omega + 2i\sqrt{\lambda}e^{X/2} - \frac{\pi}{4}i} \right] \end{aligned} \quad (20)$$

where we have made use of the asymptotic formula for $z \rightarrow \infty$

$$\begin{aligned} J_\nu(z) & \cong \sqrt{\frac{2}{\pi z}} \cos\left(z - \frac{2\nu+1}{4}\pi\right) \\ & = \frac{1}{\sqrt{2\pi z}} \left[e^{iz - i\frac{\pi}{2}\nu - \frac{\pi}{4}i} + e^{-iz + i\frac{\pi}{2}\nu + \frac{\pi}{4}i} \right] \end{aligned} \quad (21)$$

We thus find that the outgoing modes ψ_k^{out} contain both negative and positive frequency parts in the far past. This indicates particle creation.

Next we look for the correctly normalized positive frequency modes in the past ($y \rightarrow \infty$). Making the field redefinition $\psi = \sqrt{2/y} f$ in Eq. (16), we have

$$\frac{\partial^2 f}{\partial y^2} + f + O\left(\frac{1}{y^2}\right) f = 0 \quad (22)$$

Since in the far past the last term on the right-hand side is negligible, the early time vacuum modes (the correctly normalized positive frequency modes) are

$$f = \frac{1}{\sqrt{2}} e^{-iy} \quad (23)$$

which correspond to

$$\psi_k^{\text{in}} = \frac{\lambda^{-1/4}}{\sqrt{2}} e^{-\frac{X}{4} - 2i\sqrt{\lambda}e^{X/2} + i\vec{k} \cdot \vec{x}}. \quad (24)$$

The corresponding particular solution of the full equation which reduces to the above in the asymptotic limit is

$$\psi_k^{\text{in}} = \sqrt{\frac{\pi}{2i}} e^{-\pi\omega + i\vec{k} \cdot \vec{x}} H_{-2i\omega}^{(2)}(2\sqrt{\lambda}e^{\frac{X}{2}}), \quad (25)$$

where we have made use of the asymptotic formula

$$H_\nu^{(2)}(z) \sim \sqrt{\frac{2}{\pi z}} e^{-iz + i\frac{\pi}{4}(2\nu+1)}. \quad (26)$$

Comparing Eqs. (20) with (24), we can find the Bogoliubov mode mixing relations

$$\psi_k^{\text{in}} = \alpha_{\vec{k}} \psi_k^{\text{out}} + \beta_{\vec{k}} \left(\psi_{-\vec{k}}^{\text{out}} \right)^*, \quad X \rightarrow \infty \quad (27)$$

where

$$\alpha_{\vec{k}} = \frac{\lambda^{-i\omega}}{\sqrt{4\pi i\omega}} \Gamma(1 + 2i\omega) e^{\pi\omega} \quad (28)$$

$$\beta_{\vec{k}} = -\frac{\lambda^{i\omega}}{\sqrt{4\pi i\omega}} \Gamma(1 - 2i\omega) e^{-\pi\omega}. \quad (29)$$

Here we used the relation

$$\Gamma(1 + 2i\omega) \Gamma(1 - 2i\omega) = \frac{2\pi\omega}{\sinh 2\pi\omega}. \quad (30)$$

It is easy to check that the unitarity relation

$$\alpha_{\vec{k}} \alpha_{\vec{k}}^* - \beta_{\vec{k}} \beta_{\vec{k}}^* = 1 \quad (31)$$

is satisfied.

Note that in the case of our current analysis, we have a k dependence in λ , and thus the Bogoliubov coefficients have a different k -dependence than that is obtained in [15].

Incidentally, there are solutions to the equation of motion (16) with $\omega^2 < 0$; these are just the same Bessel functions as in (17) but with (real) even integer order [16]. Only the Bessel functions of the first kind are normalizable, so only they are allowed. These die off as $t \rightarrow \infty$, so they do not affect our analysis of Bogoliubov coefficients. Also, these modes have imaginary spatial momentum k , and the normalizable ones are localized near the origin. Physically, these modes represent localized transient massless string excitations.

C. Planck Distribution

Let us now calculate the spectrum of the produced particles. First, we expand the field operator $\hat{\psi}$ in terms of creation and annihilation operators a^\dagger and a associated with the positive and negative frequency modes in both the asymptotic past and future:

$$\hat{\psi} = a_{\text{out}} \psi^{\text{out}} + a_{\text{out}}^\dagger (\psi^{\text{out}})^* = a_{\text{in}} \psi^{\text{in}} + a_{\text{in}}^\dagger (\psi^{\text{in}})^*. \quad (32)$$

Substituting ψ^{in} by Eq. (27), we get

$$\alpha_{\vec{k}} a_{\text{in}, \vec{k}} + \beta_{\vec{k}}^* a_{\text{in}, -\vec{k}}^\dagger = a_{\text{out}} \quad (33)$$

$$\beta_{\vec{k}} a_{\text{in}, -\vec{k}} + \alpha_{\vec{k}}^* a_{\text{in}, \vec{k}}^\dagger = a_{\text{out}}^\dagger. \quad (34)$$

If we assume the vacuum $a^{\text{in}}|0\rangle = 0$ at $X \rightarrow \infty$, the number of strings at $X \rightarrow -\infty$ is expressed as

$$\begin{aligned} \langle 0 | a_{\text{out}}^\dagger a_{\text{out}} | 0 \rangle &= |\beta|^2 \langle 0 | a_{\text{in}} a_{\text{in}}^\dagger | 0 \rangle = |\beta|^2 \\ &= \frac{1}{4\pi\omega} \Gamma(1 - 2i\omega) \Gamma(1 + 2i\omega) e^{-2\pi\omega} \\ &= \frac{1}{e^{4\pi\omega} - 1} = \frac{1}{e^{\frac{2\pi}{\beta}k} - 1} \end{aligned} \quad (35)$$

Thus we see that the resulting spectrum has a thermal distribution with a temperature given by $\beta/(2\pi)$.

In conclusion, we have shown that cosmological fluctuation modes emerge from the tachyon condensate phase in a thermal state with temperature given by $\beta/(2\pi)$. This result is analogous to what happens in the context of string production by S-branes. The thermality is related to the fact that the tachyon condensate is periodic in imaginary time [15].

IV. CONNECTIONS TO COSMOLOGY

We have found that the spectrum of cosmological perturbations which emerges from a tachyonic big bang is a thermal spectrum rather than a vacuum. The way in which this result relates to cosmological observations obviously depends on how the initial tachyonic phase of string cosmology is connected to late time cosmology. It is usually assumed that the connection occurs through a period of cosmological inflation (see e.g. [17] for recent reviews on how inflation might arise from string theory). However, one must keep in mind that there are alternative mechanisms to create a spectrum of coherent, almost-adiabatic, nearly Gaussian, and almost scale-invariant cosmological perturbations (such as those observed). Such spectra were postulated long before inflation [18, 19, 20]. Inflationary cosmology [21] provided the first successful model which predicted such a spectrum, but recently other ideas have been proposed. In particular, it was suggested that the radiation phase of standard cosmology is preceded by a quasi-static initial Hagedorn phase [23] of perturbative string theory, and the thermal string fluctuations in this phase then lead to a spectrum of cosmological perturbations with the desired properties [24, 25] (see [26] for a recent brief review).

Let us first consider the implications of our study for the case of cosmological inflation. In this case, the modes which are probed in current cosmological observations emerged deep in the ultraviolet sea at the beginning of inflation. Our analysis provides the initial conditions for these modes, and according to our analysis the initial conditions differ substantially from the Bunch-Davies vacuum initial conditions which are usually assumed.

Let us denote the Hubble parameter at the beginning of the period of inflation by H_I . If inflation is preceded by a radiation phase (as is the case in our setup), then the temperature of the radiation at the onset of inflation is given by

$$T_I = (H_I m_{pl})^{1/2}, \quad (36)$$

where m_{pl} is the reduced Planck mass. The wavelength of a mode increases (redshifts) by a factor of

$$F = \tilde{\beta} (m_{pl}/T_I) \quad (37)$$

between the end of the tachyon phase, when the temperature is

$$T_T = \tilde{\beta} m_{pl} \quad (38)$$

(here we write the tachyon condensate gradient β of the previous sections as a dimensionless constant $\tilde{\beta}$ times the reduced Planck mass). Hence, the wavelength λ_p corresponding to the peak of the thermal distribution has a length

$$\lambda_p(T_I) = \left(\frac{H_I}{m_{pl}} \right)^{1/2} H_I^{-1} \quad (39)$$

at the beginning of the period of inflation, which is much shorter than the inflationary horizon length.

If the period of inflation has the minimal length for inflation to solve the horizon and flatness problems of standard cosmology, then between the onset of inflation and the current time the wavelength corresponding to the Hubble radius H_I^{-1} at the beginning of inflation redshifts to the length of the current Hubble radius. In this case, given our tachyon condensate precursor phase to inflation, we predict that the spectrum of cosmological fluctuations should be thermal rather than scale-invariant. Inflationary models with a much larger number of e-foldings of inflation, in which the initial inflationary Hubble radius redshifts to much more than $(m_{pl}/H_I)^{1/2}$ times the current Hubble radius by the present time, would be safe in the sense that the specific trans-Planckian signatures would have been redshifted beyond the realm of measurability of observations. Observations would only detect the far Wien tail of the thermal spectrum, which would appear scale-invariant.

In the case when our tachyonic big bang phase is the precursor phase of the quasi-static Hagedorn phase, then our study would support taking the initial conditions for fluctuations to be thermal, as assumed in [24]. In fact, it is interesting to speculate that a tachyonic big bang phase in fact is a description of the stringy Hagedorn phase. A toy model along these lines is explored in a parallel paper [27].

V. CONCLUSIONS

In this paper, we have studied the emergence of cosmological fluctuations from a tachyonic big bang phase. The result of our approximate treatment is that the cos-

mological fluctuations corresponding to zero mass sting excitations like the graviton emerge in a thermal state, with a temperature given by the tachyon profile and expected to be given by the string scale.

If the initial tachyon phase is connected to late time cosmology via a period of cosmological inflation, then our work is relevant to the question of possible trans-Planckian signatures of inflation. In fact, in models of inflation with a number of e-foldings close to the minimal number required for inflation to solve the problems of standard cosmology, we obtain a thermal rather than a scale-invariant spectrum. In models in which inflation lasts much longer, current observations would only probe the far Wien tail of the thermal spectrum, and our analysis would then lend support to the use of vacuum initial conditions for the fluctuating modes.

However, our tachyon phase can also be seen as providing the thermal initial conditions required by the recently suggested string gas cosmology structure formation scenario.

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